Unsteady Boundary Layer over a Flat Plate Started From Rest (流体力学における非定常問題)

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The paper is concerned with the motion of incompressible fluids in the boundary layer on a semi-infinite flat plate, which is started to move parallel to its length with velocity ct^n in a fluid at rest, where t is the time and $c>0$, $n \geqslant 0$. At small times the flow at a distance x from the leading edge is independent of x and described by the Rayleigh solution for an infinite plate. At large times the flow settles down to a steady state corresponding to the Blasius solution for a semi-infinite plate ($n = 0$) or approaches asymptotically to a quasi-steady state ($n>0$). A special interest centers around the problem posed by Stewartson, namely, the possibility of an essential singularity through which the transition takes place from the small-time to the large-time solution.

The analysis is simplified by using the integrated form of the boundary layer equations. A one-parameter family of velocity profiles is assumed, so that both the momentum and energy integral equations are satisfied. This results in two partial differential equations for the boundary layer thickness and the profile parameter with x and t as independent variables. The equations are hyperbolic in type, and there are two families of characteristic lines in the (x,t)-plane. The boundary conditions being specified along the x- and t-axis, the characteristic lines passing through the origin divide the plane into three regions, in each of which the solution is analytically different. The characteristic lines are thus interpreted as the location of the singularity.

On introducing $\sigma = ct^{n+1}/(n+1)x$, the partial differential equations are reduced to the ordinary differential equations with σ as a single independent variable. The solution of these equations is then obtained as a perturbation either from the small-time solution near the first characteristic line or from the large-time solution near the second characteristic line. It is found that the solution exhibits such a singularity that the leading term has a fractional exponent. Finally, an approximate solution for intermediate times is obtained, which gives results agreeing fairly well with the numerical solution by Hall.